Supersymmetry on Lattice-An Introduction

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Lattice Field Theory Course Presentation

By

Loganayagam.R.

Tata Institute of Fundamental Research, Mumbai.

General References

1. **Lattice Supersymmetry** P.H.Dondi,H.Nicolai(Nuovo Cimento **A41**(1977) 1)
2. **Lattice Supersymmetry,Superfields and Renormalisation** Joel Giedt, Erich Poppitz (hep-th/0407135)
3. **Low-dimensional Supersymmetric Lattice Models** G.Bergner, T.Kaestner, S. Uhlmann, A.Wipf (0705.2212[hep-lat])

Synopsis of this talk

1. Motivation for SUSY on Lattice
2. Problems in Lattice SUSY
3. Example : SUSY QM on Lattice
4. Lattice SUSY in d>1
5. Conclusion

Motivation for SUSY on Lattice

- SUSY QFTs : QFTs with interesting renormalisation properties (Perturbative and Non-Perturbative)
- AdS/CFT : Interesting conjecture relating quantum gravity to SYM theory.
- MSSM : Phenomenologically interesting extension of Standard Model - Non-perturbative control ? Dynamical SUSY-breaking ?

Problems in Lattice SUSY I

- Lattice explicitly breaks a sub-group of SUSY - No Poincare invariance in Lattice .
- Question : Is it enough to have SUSY closing on discrete translations ?
- Answer : Not good enough .
- No Leibnitz rule on lattice $\Longrightarrow$ SUSY algebra might not extend to products of fields

Problems in Lattice SUSY II

- SUSY QFTs often have chiral fermions
- Fermion doubling can destroy SUSY (Doublers might not have bosonic superpartners)
- If SUSY is broken in the Lattice, then SUSY violating non-irrelevant operators should be fine-tuned away.
- Can we find Efficient methods to do the fine-tuning ?

Problems in Lattice SUSY III

- SUSY does more than just forbidding certain interactions
- It actually constrains various interactions (Example : Masses of super-partners, different couplings etc.)
- How do we impose such constraints ? Will some other local/non-local Lattice symmetry work ?

SUSY QM-Continuum Version

$$S_{\text{Continuum}} = \int d\tau \left(\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}W'^2 + \bar{\psi}\dot{\psi} + \bar{\psi}W''\psi \right)$$

Under SUSY transformations :

$$\delta\phi = \bar{\epsilon_1}\psi + \bar{\psi}\epsilon_2 \; ; \; \delta\bar{\psi} = -\bar{\epsilon_1}(\dot{\phi} + W') \; ; \; \delta\psi = \epsilon_2(\dot{\phi} - W')$$

$$\delta S_{\text{Continuum}} = \int d\tau \frac{d}{d\tau}\left(\dot{\phi}\bar{\psi}\epsilon_2 - W'\bar{\epsilon_1}\psi \right)$$

SUSY QM on Lattice -Naive Discretisation

Which Lattice derivative to use ?

- Forward Derivative : $\partial^f_{xy} \equiv \frac{\delta_{x+1,y} - \delta_{x,y}}{a}$
- Backward Derivative : $\partial^b_{xy} \equiv \frac{\delta_{x,y} - \delta_{x-1,y}}{a}$
- Symmatrised Derivative : $\partial^s_{xy} \equiv \frac{\delta_{x+1,y} - \delta_{x-1,y}}{2a}$
- Wilson Derivative : $\partial^W_{xy} \equiv \partial^s_{xy} - \frac{ar}{2}\Box_{xy}$

Where the Lattice Laplacian $\Box_{xy} \equiv \frac{\delta_{x+1,y} - 2\delta_{x,y} + \delta_{x-1,y}}{a^2}$

SUSY QM on Lattice -Naive Discretisation(contd.)

- Bad News: Naive discretisation of $S_{\text{Continuum}}$ generically leads to **No SUSY** in the continuum.
- ∂^b for bosons, ∂^W for Fermions gives

$$m_{bos}(a) = (139.52 \pm 8.45)a + (12.23 \pm 0.08)$$

$$m_{ferm}(a) = -(186.25 \pm 4.98)a + (18.40 \pm 0.05)$$

(From Bergner et.al.[3], Linear interpolation from Monte Carlo simulations)

Preserving SUSY on Lattice

- Can we do any better ?
- Dondi & Nicolai [1] : Discretise more carefully using superspace
- Discretise bosonic co-ordinates , but leave fermionic co-ordinates continuous

$$\int d^n x \int d\theta d\bar{\theta} \longrightarrow \sum_n \int d\theta d\bar{\theta}$$

- Still, Leibnitz Rule is a problem - How should SUSY act on products ?

Preserving SUSY on Lattice II

- Using methods of Nicolai, we can write down a better action for SUSY QM

$$S_{\text{susy}} = S_{\text{naive}} + a \sum (\partial^b \phi) W'$$

- This preserves half the supersymmetry

$$\delta_1 \phi = \bar{\epsilon_1} \psi \; ; \; \delta_1 \bar{\psi} = -\bar{\epsilon_1} (\partial^b \phi + W') \; ; \; \delta_1 \psi = 0$$

- We get back the other SUSY transformation in the continuum limit

Lattice SUSY in $d > 1$

- Similar methods as above can be employed in discretising SUSY in higher dimensions
- Typically, we want to exactly implement atleast some amount of SUSY on the Lattice and get the remaining SUSY by fine-tuning
- Such methods have been quite successful in 1+1 d (eg., Lattice Wess-Zumino Models)
- In dimensions higher than 2, the picture is not very clear ..

Conclusion

- I have tried here to introduce the basic issues in implementing SUSY on Lattice
- For a more comprehensive review of the various ideas that have been proposed in this field - see the References that follow
- It seems difficult to write down SUSY Lattice QFTs - We probably need new ideas...

Useful Reviews

1. **Advances and Applications of Lattice Supersymmetry** Joel Giedt(hep-lat/0701006)
2. **DEconstruction and other approaches to Supersymmetric Lattice Field Theories** Joel Giedt(hep-lat/0602007)
3. **Predictions and Recent results in SUSY on Lattice** Alessandra Feo(hep-lat/0410012)