Supersymmetry on Lattice-An Introduction

28\textsuperscript{th} June 2007
Lattice Field Theory Course Presentation

By
Loganayagam.R.
Tata Institute of Fundamental Research, Mumbai.
1. **Lattice Supersymmetry**  
   P.H. Dondi, H. Nicolai (Nuovo Cimento A41 (1977) 1)

2. **Lattice Supersymmetry, Superfields and Renormalisation**  
   Joel Giedt, Erich Poppitz  
   (hep-th/0407135)

3. **Low-dimensional Supersymmetric Lattice Models**  
   G. Bergner, T. Kaestner, S. Uhlmann, A. Wipf  
   (0705.2212[hep-lat])
Synopsis of this talk

1. Motivation for SUSY on Lattice
2. Problems in Lattice SUSY
3. Example : SUSY QM on Lattice
4. Lattice SUSY in d>1
5. Conclusion
Motivation for SUSY on Lattice

- SUSY QFTs: QFTs with interesting renormalisation properties (Perturbative and Non-Perturbative)
- AdS/CFT: Interesting conjecture relating quantum gravity to SYM theory.
- MSSM: Phenomenologically interesting extension of Standard Model - Non-perturbative control? Dynamical SUSY-breaking?
Problems in Lattice SUSY I

- Lattice explicitly breaks a sub-group of SUSY - No Poincare invariance in Lattice.
- Question: Is it enough to have SUSY closing on discrete translations?
- Answer: Not good enough.
- No Leibnitz rule on lattice $\implies$ SUSY algebra might not extend to products of fields.
Problems in Lattice SUSY II

- SUSY QFTs often have chiral fermions
- Fermion doubling can destroy SUSY (Doublers might not have bosonic superpartners)
- If SUSY is broken in the Lattice, then SUSY violating non-irrelevant operators should be fine-tuned away.
- Can we find Efficient methods to do the fine-tuning?
Problems in Lattice SUSY III

• SUSY does more than just forbidding certain interactions

• It actually constrains various interactions (Example: Masses of super-partners, different couplings etc.)

• How do we impose such constraints? Will some other local/non-local Lattice symmetry work?
SUSY QM-Continuum Version

\[ S_{\text{Continuum}} = \int d\tau \left( \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} W'^2 + \bar{\psi} \dot{\psi} + \bar{\psi} W'' \psi \right) \]

Under SUSY transformations :

\[ \delta \phi = \epsilon_1 \psi + \bar{\psi} \epsilon_2 ; \quad \delta \bar{\psi} = -\bar{\epsilon}_1 (\dot{\phi} + W') ; \quad \delta \psi = \epsilon_2 (\dot{\phi} - W') \]

\[ \delta S_{\text{Continuum}} = \int d\tau \frac{d}{d\tau} \left( \dot{\phi} \bar{\psi} \epsilon_2 - W' \bar{\epsilon}_1 \psi \right) \]
SUSY QM on Lattice - Naive Discretisation

Which Lattice derivative to use?

- **Forward Derivative**: \( \partial^f_{xy} \equiv \frac{\delta_{x+1,y} - \delta_{x,y}}{a} \)
- **Backward Derivative**: \( \partial^b_{xy} \equiv \frac{\delta_{x,y} - \delta_{x-1,y}}{a} \)
- **Symmatrised Derivative**: \( \partial^s_{xy} \equiv \frac{\delta_{x+1,y} - \delta_{x-1,y}}{2a} \)
- **Wilson Derivative**: \( \partial^W_{xy} \equiv \partial^s_{xy} - \frac{ar}{2} \Box_{xy} \)

Where the Lattice Laplacian \( \Box_{xy} \equiv \frac{\delta_{x+1,y} - 2\delta_{x,y} + \delta_{x-1,y}}{a^2} \).
Bad News: Naive discretisation of $S_{\text{Continuum}}$ generically leads to No SUSY in the continuum.

∂ for bosons, $\partial^W$ for Fermions gives

$$m_{\text{bos}}(a) = (139.52 \pm 8.45)a + (12.23 \pm 0.08)$$

$$m_{\text{ferm}}(a) = -(186.25 \pm 4.98)a + (18.40 \pm 0.05)$$

(From Bergner et.al.[3], Linear interpolation from Monte Carlo simulations)
Preserving SUSY on Lattice

- Can we do any better?
- Dondi & Nicolai [1]: Discretise more carefully using superspace
- Discretise bosonic co-ordinates, but leave fermionic co-ordinates continuous

\[
\int d^n x \int d\theta d\bar{\theta} \longrightarrow \sum_n \int d\theta d\bar{\theta}
\]

- Still, Leibnitz Rule is a problem - How should SUSY act on products?
Preserving SUSY on Lattice II

• Using methods of Nicolai, we can write down a better action for SUSY QM

\[ S_{\text{susy}} = S_{\text{naive}} + a \sum (\partial^b \phi) W' \]

• This preserves half the supersymmetry

\[ \delta_1 \phi = \bar{\epsilon}_1 \psi ; \quad \delta_1 \bar{\psi} = -\bar{\epsilon}_1 (\partial^b \phi + W') ; \quad \delta_1 \psi = 0 \]

• We get back the other SUSY transformation in the continuum limit
Lattice SUSY in $d > 1$

- Similar methods as above can be employed in discretising SUSY in higher dimensions.
- Typically, we want to exactly implement at least some amount of SUSY on the Lattice and get the remaining SUSY by fine-tuning.
- Such methods have been quite successful in $1+1$ d (e.g., Lattice Wess-Zumino Models).
- In dimensions higher than 2, the picture is not very clear..
Conclusion

• I have tried here to introduce the basic issues in implementing SUSY on Lattice

• For a more comprehensive review of the various ideas that have been proposed in this field - see the References that follow

• It seems difficult to write down SUSY Lattice QFTs - We probably need new ideas...
Useful Reviews

1. Advances and Applications of Lattice Supersymmetry Joel Giedt(hep-lat/0701006)

2. DEconstruction and other approaches to Supersymmetric Lattice Field Theories Joel Giedt(hep-lat/0602007)

3. Predictions and Recent results in SUSY on Lattice Alessandra Feo(hep-lat/0410012)